

This tutorial is going to deal with an important statistical concept called the law of large numbers. So let's take a look. Suppose we flip a coin and determine the long-term relative frequency of heads. We're going to determine heads to be a win. So if we start here, we're going to say if we flip heads, we're going to go up to the next dot from this segment. And if we flip tails, we're going to go down to the lose dot from this segment.

So suppose the first coin I flip is heads. I'll go up to the first dot. And I am as high as I can go. So if I flip heads again, I'm only going to just go to the second dot. Suppose the third time, I flip tails. That means I'm going to follow that down. And then I flip heads again, I'm going to follow it back up. And so I can continue looking at what happens in the long term by simulating the rest.

And what you start to notice is these swings at the beginning are pretty wild swings. They're very large shifts, whereas down over here, these are not very big of shifts. And what you seem to be noticing is that the long term relative frequency of heads seems to be settling in right around 50%.

The law of large numbers says that the more times you run a chance experiment, the relative frequency of an event approaches the true probability of that event. And you're going to get closer and closer to the right answer the larger the number of trials becomes. So we can keep going, and we would probably get closer and closer and closer to 50% as we kept going to the right.

Now, this isn't to say that odd things don't happen. Look closely. We did have runs of three and four heads during our experiments. So it's not to say that things don't happen that are unusual. Four heads in a row is fairly unusual, but it happens. We also had a couple of different runs of three tails in a row. So unusual things can happen in the short term, but we know predictably what will happen in the long term is that this blue line will start to settle in right around 50%.

Now, you might hear things like this. On the radio, you might hear a sports announcer say something like, he hasn't gotten a hit in his last four at-bats, so he's due to get a hit. Or you might hear something like this. He's gotten a hit in his last four at-bats. He's on a hot streak. He'll certainly get a hit this time too. Well, both of these have reached the same conclusion, but for opposite reasons.

This first saying is saying that he's due, which means he's going to get a hit this time because he hasn't gotten a hit so far. This one is saying that he's going to get a hit this time because he has gotten a hit. None of these make any sense. Both of these logically are fallacies. They apply the law of large numbers, which means that maybe this player gets a hit one out of every three times he's at bat in the long term.

But what they're trying to do is apply that law of large numbers to these five at-bats that this player has had. So you can't apply the law of large numbers to short term events, because streaks like we saw in the coin flipping example are fairly common. Five trials is not considered a large number of trials. You can't say that things will happen over the short term. You can only say what predictably will happen over the long term.

And it's important to understand the difference between short term average and long term average. This is sometimes called the law of averages. And it's not actually a mathematical term. It's a psychological game people play with themselves to convince themselves that favorable outcomes are just over the horizon.

Where you see a lot of this is in the casino. People will say, oh, I've won five times in a row. I can't lose. I'm on a hot streak or I'm playing with house money. These are all very common phrases that you can hear in the casino. Meanwhile, you have people say, oh, I've lost five times in a row. I must be due for a win. The law of averages, the false law of averages, it's also called the gambler's fallacy or the gambler's ruin. People can convince themselves that favorable outcomes are about to happen even when they're not necessarily going to happen, because they're applying the law of large numbers to small numbers of trials.

So to recap, the law of large numbers is super important. It says that over the long term, the relative frequency of an event is the probability of that event. And the law of large numbers sometimes will be inappropriately applied to short term events. Sometimes that's called the law of averages and the gambler's fallacy. And it's important not to fall into that trap of applying the predictable nature of large numbers to the unpredictable nature of short term events. So we talked about the law of large numbers and we also talked about how that differs from the law of averages, which is also called the gambler's fallacy and gambler's ruin. Good luck and we'll see you next time.