
Hi. This tutorial covers a type of normal distribution called the standard normal distribution. So let's just start by remembering what a normal distribution is first. So remember that a normal distribution is a single-peaked, bell-shaped, symmetric distribution. And remember that all normal distributions have special properties, one of them being the 68-95-99.7 rule, which we're going to use in a little bit here.

Now, a standard normal distribution is, again, a type of normal distribution. But a standard normal distribution has a mean equal to 0 and a standard deviation equal to 1. So if we notate that, μ would equal 0. σ would equal 1. Or if you're dealing with a sample, \bar{x} would be 0, and s would be 1.

Any normal distribution can be converted to a standard normal distribution using z-scores. And remember, the formula for a z-score for dealing with a population is x minus μ over σ . OK.

So let's take a look at an example here and see if we can start utilizing a standard normal distribution. So the heights of adult human males are normally distributed with a mean of 70 inches and a standard deviation of 3 inches. So let's draw the distribution, then convert it to a standard normal distribution.

All right. So I'm going to make a nice, big distribution here. So it's normal, has that bell shape to it. What I'm going to do is I'm going to call this axis the x-axis. x is going to represent my height data. So it's centered at 70.

One standard deviation would be 73. Two standard deviations above the mean would be 76. And three above would be 79-- just adding that 3 each time. So we know that σ was [INAUDIBLE].

Now let's go below the mean. So this would be 67, 64. And I'll just extend it so it goes to 61. OK. So that right there is a normal distribution.

Now, to convert it to a standard normal distribution is basically just adding another axis onto the bottom of that. And this is going to be the z-axis. And these are really just going to represent the z-scores for these kind of landmark values. So remember that z-score is x minus μ over σ .

So if we want the z-score for the mean, it'd be 70 minus 70 over 3. 70 minus 70 is 0, divided by 3 is 0. So the z-score for the mean is always 0.

73-- so it would be 73 minus 70, which would be 3, divided by 3 is 1. 76-- 76 minus 70 is 6, divided by 3

is 2. You might guess this would be 3. OK. Then this should make sense. A z-score basically tells you how many standard deviations you are from the mean. If it's positive, it's above the mean.

Now, 67-- 67 minus 70 is negative 3, divided by 3 is negative 1. 64 minus 70 is negative 6, divided by 3 is negative 2. And this would be negative 3. So what we just did is standardized a normal distribution using z-scores.

Now, we'll come back to this picture in a second, but let's see if we can answer some questions now based on that distribution. So number one, what percent of males are shorter than 67 inches? So 67 inches.

So we know 67 inches is here with a z-score of negative 1. So we want to know, well, what percent of men have a height less than that.

OK. So if we think about the empirical rule, remember, the empirical rule tells us 68% of the data is within one standard deviation. So if 68% of the data are between these two values, that means 32% of the data have to be split between these two tails. Since the distribution is symmetric, 16% would be on this tail. 16% would be in this tail. So might the answer to that question would be 16%, using that 68-95-99.7 rule.

So to answer this question, we know that z was equal to negative 1. And the percent was 16%.

Now question two-- what percent of males are shorter than 65 inches. Now, that's going to be a little trickier. So if you think about 65, 65 isn't one of these important landmarks. So we can't really use the 68-95-99.7 rule here.

So what we're going to do instead is we're going to start by calculating a z-score for 65. So it'd be 65 minus 70 divided by 3. And going to the calculator, we would do-- that gives me negative 5 divided by 3 is negative 1.667.

OK. So again, we said that we can't use our general rule for normal distributions. But what we can instead is use what's called a z-table. So z-table is a table that gives the percent of values to the left of a given z-score on a normal distribution.

So remember, we're trying to figure out, well, what percent of the data lies to the left of a z-score of negative 1.667. So on a z-table-- this is kind of what a z-table usually looks like-- you can see up at the top, what's shaded in here is a probability or a percent. And this boundary is your z boundary.

So we're looking for probability to the left of a z-score of negative 1.67. I'm just going to round it to negative 1.67 this time. So what I do is I look for my first two digits on this side. So this page has all of the negative values. There's another page that has all the positive z-score values.

So what we're going to do is first look for negative 1.6. So that gives me the 1 and the 6 part. And then up along the top of my table, I'm going to use these values to represent the hundredths place.

So I need to look for 07 here. So basically what I'm going to do then is follow this down until I get to negative 1.6, which is here. And that's going to give me this probability right there, which ends up being about 0.0475. OK. So the answer to this question, then, would be 4.75%. So there's about-- a little less than 5% of males are shorter than 65 inches.

OK. We're going to use the z-table for a couple more problems just to make sure we're familiar with this now. So what percent of males are taller than 72 feet-- or excuse me-- 72 inches, which is 6 feet.

So first thing you have to do is you have to calculate a z-score. So we'd take 72 minus 70 divided by 3. So this will end up giving me $\frac{2}{3}$. So 2 divided by 3 is now just 0.67. I'll just round it to the nearest hundredth again.

But now I'm looking for taller. The process is still going to be pretty similar. So what I'm going to do now is I'm going to go to the positive z-scores. And I'm going to find where 0.067 is. So I look for a 0.6 over here, which is here. And then I'm going to follow that over to 0.07. So this is going to be 0.7486-- so 0.7486.

OK. But now remember what this value tells me is what's the-- this is the probability of somebody being shorter than 72 inches. So what I want to do with this value now is I want to take-- if this is a percent, it's about 75%.

So what I want to do is I take 100% minus this as a percent. So I'm going to take 100% minus 74.86%. So again, doing that on the calculator, 100 minus 74.86, that's going to give me about 25.14. So this should be 25.14%. So this would be the answer to that question.

So if you're answering a greater than question, what you're going to have to do is you're going to have to take 100% minus whatever that less than probability is. All right. And this is a problem that I will let you do on your own.

And that's been your tutorial on the standard normal distribution. Thanks for watching.