Hi, my name is Anthony Varela. And in this tutorial, I'm going to talk about rationalizing the denominator. So first, we're going to take a look at irrational denominators, what those look like, and why we don't like them in mathematics. Then we're going to talk about conjugates, and how we can use what's called conjugate to rationalize the denominator. And then we're going to bring everything together and go through an example where we rationalize a denominator.

So let's start off by talking about irrational denominators. What do I mean? Well, here's an example of a fraction that has an irrational number in the denominator. And in mathematics, we consider this to be unsimplified.

And this is actually because back when calculators weren't around, you can imagine how cumbersome it was or hard to divide a number by an irrational number because it has an ongoing, never-ending decimal pattern, which is just, to be honest, a pain. So what we like to do then is simplify this fraction and rewrite it so that's equivalent, but the denominator is a rational number instead.

So thinking about how I can go from the square root of 2, which is irrational, to something that is rational, I might multiply the square root of 2 by itself. That will give me the integer 2, which is a rational number. But I am writing an equivalent fraction, so I have to multiply both the denominator and the numerator by the same value.

So you notice here that we're multiplying this fraction by a quantity that's equal to 1. It might not look like 1, but it has the same quantity in the numerator and denominator. So this equals 1. So multiplying by 1 doesn't change the value, but it is going to change how this looks.

So now I can multiply across my numerators. And I have an equivalent fraction here that reads 3 times the square root of 2 over 2. It has the same exact value-- just that we have a rational number in our denominator.

So now I'm going to talk about conjugates, which helps us rationalize denominators. But first, we have to understand what a conjugate is. So here's an expression, the square root of six plus 5. And the conjugate of this expression is the square root of 6 minus 5.

Let's take a look at another example. The square root of 3 minus 7. And the square root of 3 plus 7 are conjugates. So do you think that you can draft up your own definition of what a conjugate might be?

So a conjugate of a binomial-- that just means something has two terms, one term and another term-- the conjugate of a binomial is a binomial with the opposite sign between terms. So these look nearly identical. The only difference is we have gone from a plus to a minus. And in here, we have gone from a minus to a plus.

So thinking about my example before, we had just one term, the square root of 2. And this can be considered its own conjugate. So the square root of 2 and the square root of 2 are conjugates of each other. One way to think about this would be the square root of 2 plus 0 and the square root of 2 minus 0.

So how do conjugates help us rationalize denominators? So we're going to go through this example here-- the square root of 2 minus 2 over the square root of 6 minus 2. And for now, I'd like to focus on just our denominator. And we're going to be using the conjugate of the square root of 6 minus 2 to rationalize this expression, which is the denominator of this fraction.

So first, what is the conjugate of the square root of 6 minus 2? Well, that would be the square root of 6 plus 2. And we're going to be multiplying these two expressions together. And you notice we can do this using FOIL.

So recall that with FOIL, we are multiplying the first two terms, then the outside terms, then the inside terms, and then the last terms here. So multiplying our first terms, this would be the square root of 6 times the square root of 6. That just gives us the integer 6.

Well, next, we're going to multiply the outside terms. That would be the square root of 6 times 2. So that's plus 2 times the square root of 6. Now I'm going to multiply my inside terms, which is negative 2 times the square root of 6. So I'll show subtracting 2 times the square root of 6.

Then multiplying my last two terms, I have negative 2 times positive 2, which is a negative 4. Well, notice that I have a positive 2 times the square of 6 and a negative 2 times square root of 6. Those are like terms that I can combine, but they combine to 0. So I really have just 6 minus 4. And 6 minus 4 is 2.

So you see that multiplying this square root of 6 minus 2 by its conjugate gives me an expression that has no radical at all. So it rationalizes that expression. And just as a reminder, if you had just a single radical like our very first example, the square root of 2, you can multiply that by a conjugate, which it, itself, the square root of 2. And you also no longer have a radical.

So multiplying radical conjugate rationalizes the expression. And that's the whole goal here with rationalizing the denominator. So let's return to our original example, and we're going to go through the full process of rationalizing the denominator.

So I know that I have to multiply this expression by a quantity equal to 1 to keep its value the same. And I know I want to use the conjugate of that denominator. So I need to have square root of 6 plus 2 in my denominator and in my numerator. So we're going to be multiplying these two fractions, and we're going to be using FOIL to do this.

So let's start with going through our denominator. We've already gone through it, but we'll see the steps again. Multiplying the first two terms, multiplying the outside terms, multiplying the inside terms, and then multiplying the last terms. Now we're going to do the same thing FOIL, but with our two numerators.

So multiplying the first two terms in our numerator, we get the square root of 12. Multiplying our outside terms in our numerators, we get plus 2 times the square root of 2. Multiplying those two inside terms, we get negative 2 times the square root of 6. And then multiplying the last terms, negative 2 and positive 2 gives us negative 4.

So this looks very messy, but all we want to do now is consider our numerator and our denominator separately, combine any like terms, and do some simplification. Well, we already know from our example before, that our denominator simplifies to 2. So I'm kind of skipping this step. But you can, of course, rewind to see how I got that.

So now let's go ahead and simplify our numerator. And I'm going to start with-- I see we have an integer there. I'm going to start with that right away, negative 4. And I'm actually going to skip the square root of 12 for now. We'll talk about that in a bit.

So I'm going to write down my plus 2 times the square root of 2 minus 2 times of square root of 6. And now this is how I'd like to write the square root of 12. I'm writing that as 2 times the square root of 3.

And if you're wondering how I did that, I took the number 12, and I broke it down into 4 times 3. So the square root of 12 equals the square root of 4 times the square root of 3. The square root of 4 is 2. So that's how I got this expression right here.

So we notice that we've achieved our goal. Our denominator has been rationalized. But we can actually go one step further. And it's not always the case, but in this example, we can simplify more. So here's our expression so far.

And I notice that every term in my numerator has a factor of 2 in it. So I'm going to factor that out-- so factoring out the common factor of 2. So here's my factor of 2. And in parentheses, I've divided everything by 2 here.

And now I notice that there's a common factor in my numerator and my denominator. Those cancel. So now I'm left with an expression that doesn't even look like a fraction at all, which is kind of cool. But we can think about our denominator as being one, which I don't always have to write. So we've rationalized our denominator.

So let's review today's tutorial. The conjugate of a binomial is a binomial with opposite signs between terms. So a plus b and a minus b are conjugates. We used FOIL to multiply two binomials. The process is multiplying the first terms, the outside terms, the inside terms, and then the last terms.

And remember, that multiplying radical conjugates rationalizes the expression, which is the goal of rationalizing the denominator. So thanks for watching this tutorial on rationalizing the denominator. Hope to see you next time.