Hi, and welcome. This is Anthony Varela and today we're going to solve absolute value equations. So we're going to start with a simple example where we just have the absolute value of x equals some value. Then we're going to see a linear expression inside absolute value bars and talk about how to solve those types of equations. And then our last example, will have an expression inside of an absolute value bar and then something outside of it too on the same side of the equals sign.

So first, let's review absolute value. Now, recall that the absolute value of a number is its distance away from 0 on the number line. So if we put 4 on the number line, you would see that 4 is four units away from 0 on the number line.

And on the other side of 0, we can see that negative 4 is also four units away from 0 on the number line. So we could say then that the absolute value of 4 equals 4, and the absolute value of negative 4 equals 4 as well. Now I'd like to pay particular attention to this absolute value of negative 4.

So if we think about negative 4 being inside of the absolute value bar, we had to take the opposite of that in order to express it as positive 4. So the opposite of negative 4 is positive 4, and that's the absolute value.

So we get into an interesting definition then of absolute value. And this is what we call the piecewise definition to absolute value. And it's OK if you don't quite understand piecewise functions. But essentially, this just means that we're evaluating this expression, the absolute value of x, differently depending on what x is.

And now when x is greater than or equal to 0, we don't really do anything. We can erase the absolute value bars and we'd be fine. So the absolute value of x is x when x is greater than or equal to 0.

Now, when x is less than 0, so when x is negative, then we evaluate it as the opposite of that. So that would be this example right. Here we had a negative value inside the absolute value bars. So we took the opposite of that, which would be positive 4. So we're going to be using this piecewise definition of absolute value to solve some absolute value equations.

So we'll start off with a simple example, the absolute value of x equals 5. Now in order to solve this equation, we're going to make two different equations out of this from our piecewise definition here. So one of the equations is going to be x equals 5. And the other is going to be negative x equals 5.

So when we took away the absolute value bars, we were able to just keep it as it is minus the absolute

value bars. And then we also the second equation where we're taking the opposite of this expression, so negative x equals 5.

Well, we already have then one of our solutions, x equals 5. If negative x equals 5, then x equals negative 5. So these are two solutions then to this absolute value equation. x could be 5, x could also be negative 5. When you take the absolute value, we get 5.

So putting this on a graph-- So here's a graph of the absolute value of x. And then we're going to have y equals 5 here. So the intersections then to y equals the absolute value of x and y equals 5 are at negative 5 and positive 5.

So now let's get a little bit more complicated. Here we have a linear expression inside of our absolute value bars. So how can we solve the absolute value of 2x minus 3 equals 3? Well, we're going to create, once again, two different equations.

Our first equation is going to be 2x minus 3, we're just erasing the absolute value bars. But our other equation we're going to take the negative of the entire expression. So it's very important that I'm grouping my mx plus b in parentheses and putting a negative out in front.

So let's go ahead and work with our first equation, 2x minus 3 equals 3. Well, I'll add 3 to both sides of the equation and then divide by 2. So one of my solutions is x equals 3. Well, my other equation is going to be negative 2x minus 3 equals 3.

And so first what I'm going to do is distribute that negative throughout 2x minus 3. So we have negative 2x plus 3 equals 3. And to solve for x, I am going to subtract 3 from both sides. So if negative 2x equals 0, then x just equals 0.

So I have my two solutions, x equals 3 and x equals 0. Let's go ahead and take a look at this on the graph. So this is y equals the absolute value of 2x minus 3. And now I'm also going to plot y equals 3 and look at our points of intersection. Those occur at x equals 0 and x equals 3, our solutions for x here.

One final example, here we have an absolute value expressions here, so we have the absolute value of negative x plus 2. And then we have something outside of the absolute value bars, but it's still on the side of the equals sign that contains the absolute value. So how do we deal with something like this?

Well, we'd like to remove it from that side. So we want to have only the absolute value expression on

that side of the equation. So I'm just going to subtract 1 from both sides of the equation. And so I'm going to move it to the other side.

So really, I am dealing with an equivalent equation, the absolute value of negative x plus 2 equals 3. And this fits this form. So it's just like the previous example.

So one of our equations is going to be negative x plus 2 equals 3. And then I'm going to group negative x plus 2 and apply a negative, so taking the opposite of this expression. That also equals 3.

So solving for negative x plus 2 equals 3, I'll subtract 2 from both sides. And if negative x equals 1, then x equals negative 1. That's one of our solutions.

For this equation here, first what I'll do is distribute that negative. So I have a positive x minus 2 equals 3. And then I'll add 2 to both sides of the equation. And the other solution is x equals 5.

Let's once again confirm this on the graph. So this is the graph of y equals the absolute value of negative x plus 2 plus 1. And then I have y equals 4. And our points of intersection occur at negative 1 and 5. Those were our solutions to x

So let's review our notes on solving absolute value equations. We talked about this piecewise definition of absolute value. When x is greater than or equal to 0, we can just evaluate the absolute value of x as x. If x is a negative, so that would be less than 0, then we have to take the opposite of what we see in the absolute value bars.

So when solving absolute value equations, you're taking the absolute value equation and you're really making two separate equations. And we saw that in all of our examples. One equation you're just are racing the absolute value bars.

But in your other equation, your grouping in that entire expression and taking the opposite of that, or multiplying it by negative 1. So thanks for watching this tutorial on absolute value equations. Hope to see you next time.